

On the mechanism of collisionless damping of sound in dilute Bose gas with condensate

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We develop a microscopic theory of sound damping due to Landau mechanism in dilute gases with a Bose condensate. It is based on the coupled equations that take into account the mutual influence of condensate density, phase (or superfluid velocity), and distribution function of elementary excitations on evolution of the system. These equations have been derived in earlier works within a microscopic approach which employs the Peletminskii-Yatsenko reduced description method for quantum many-particle systems and Bogoliubov model for a weakly nonideal Bose gas with a separated condensate. The dispersion equations for sound oscillations are obtained by linearization of the mentioned evolution equations in the collisionless approximation. They are analyzed both analytically and numerically. The expressions for speed of sound and decrement factor of sound are obtained in the limiting cases of high and low temperatures. It is shown that at low temperatures, the decrement of collisionless damping of sound has a quadratic dependence on temperature. Such dependence essentially differs from that obtained earlier by other authors in phenomenological approaches. It is also demonstrated that at high temperatures, the sound damping coefficient has a linear dependence on temperature that coincides with earlier results of other authors. We discover a finite temperature at which the speed of sound is exactly equal to the speed of sound at zero temperature. This effect is due to non-analytic dependence of dispersion characteristics of the system on temperature at this point. We also indicate the possibility of oscillations for the decrement or increment coefficients of sound in the vicinity of the discovered temperature.

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I. INTRODUCTION

The study of mechanisms of sound damping in a Bose-Einstein condensate (BEC) has a long history. Calculation of the sound damping coefficient in systems with BEC is a rather complicated theoretical problem. First expressions for damping rate in such systems have been apparently obtained in [1, 2] for the spatially homogeneous case.

The direct experimental observation of BEC [3–5] has simulated a great number of works devoted to various aspects of this phenomenon (see in this regard, for example, [6, 7] and references therein). A number of papers, both theoretical and experimental, deal with the problem of propagation and damping of excitations in Bose gases with the presence of a condensate [8–16].

It is currently assumed that the Landau damping is the most probable mechanism of sound relaxation in the so-called trapped Bose condensates. This mechanism consists in collisionless absorption of the energy of oscillations by quanta of elementary excitations [12, 13]. In this regard it is worth recalling that the existence of specific collective excitations in a gas with BEC is known since the pioneering work of Bogoliubov [17]. In this pa-

per, a special perturbation theory has been proposed for a weakly non-ideal and spatially homogeneous Bose gas with a condensate in which the repulsive interaction acts between atoms. This theory predicts the elementary excitation spectrum for such system at zero temperature. At small wave vectors, it coincides with the spectrum of sound oscillations in a condensed Bose gas.

The first work, which proposed a method for calculating the sound damping rate in trapped BEC due to Landau mechanism, is apparently a paper [12] (see also [13]). The approach developed by the authors uses the perturbation theory and is based on the calculation of the difference in probabilities between emission and absorption of quanta of oscillations by elementary excitations in the system. It should be noted that the trapped condensates represent a spatially inhomogeneous system. This fact essentially complicates the analytical calculations. The method of Ref.[12] has been shown to be suitable for numerical calculations of sound damping in a trapped condensate (see the same paper). For a spatially homogeneous BEC, this method has provided analytical formulae for damping according to Landau mechanism [12]. In this case the authors have reproduced the results obtained in Refs. [1, 2].

It should be stressed that the formulae of Ref. [12] (and, hence, the results of Refs.[1, 2]) can be obtained by other method involving the kinetic equation for distribution function of elementary excitations (in this connection see, e.g., Ref. [6]). In other words, the semi-phenomenological approach to calculation of the damping coefficient shown in Ref. [6] is equivalent to the method of Ref. [12]. This is *inter alia* indicated in Ref. [12]. Thus, the correctness of the problem of sound damping in BEC which is under solution becomes dependent on the type of kinetic equation for elementary excitations. Therefore, we must be confident of correctness of kinetic equation which is used to solve the mentioned problem. However, it is clear that in the most general case the only use of kinetic equation for excitations is not sufficient. The system should be described by the coupled evolution equations which take into account the mutual influence of condensate density, phase (or superfluid velocity) and distribution function of elementary excitations. Thus, there is a necessity for consistent (controlled) derivation of the coupled equations describing the system with BEC at a particular stage of its evolution. The consistent derivation of the system of coupled equations can be achieved using a microscopic approach, proceeding from the first principles. This problem has been solved in [10, 18, 19, 21] within a microscopic approach based on the reduced description of quantum many-particle systems and Bogoliubov model for a weakly non-ideal Bose gas with condensate [17]. Note that the foundation of the reduced description method for classical (not quantum) systems has been formulated by N. Bogolyubov [22]. This method has been extended to quantum many-particle systems by S. Peletinskii and coauthors (see Ref. [23]). The synthesis of approaches elaborated in Refs. [17] and [23] has allowed one to obtain the kinetic equation for distribution function of elementary excitations coupled with the evolution equations for condensate density and superfluid momentum [18, 19]. The validity of such system of equations is confirmed by its controlled derivation within the framework of special perturbation theory in weak interparticle interaction. In addition, the following fact says in favor of the mentioned coupled equations: they have been employed to derive hydrodynamic equations of a superfluid in which the smallness of the difference between superfluid and normal velocities is not taken into account [20, 21]. As this difference tends to zero, the obtained equations are reduced to the well-known hydrodynamic equations of Khalatnikov (see e.g. [24]).

Notwithstanding the above considerations, the equations of Ref. [19] have not yet been used to study the propagation and damping of sound in a dilute gas with BEC. However, the same considerations allow us to hope that the correct solution of evolution equations found in Ref. [19] should lead us to the correct expression for the sound damping coefficient in a gas with BEC. The present paper is devoted to the study of collisionless mechanism of sound damping in dilute gases with BEC

on the basis of general dynamic equations of such systems obtained in [19] from the first principles. As will be seen later, the results obtained in the present work significantly differ in some cases from those of Refs. [1, 2] and, consequently, of Ref. [12]. For example, in the present study it is shown that in the collisionless approximation the sound damping factor at low temperature is proportional to the square of the temperature $\gamma \sim T^2$, whereas the results of [1, 12] give $\gamma \sim T^4$ dependence.

II. THE KINETICS OF SPATIALLY INHOMOGENEOUS BOSE GAS IN THE PRESENCE OF CONDENSATE

In the construction of the kinetics of spatially inhomogeneous Bose gas the authors [19] started from the Liouville equation for the statistical operator $\rho(t)$

$$i\frac{\partial\rho(t)}{\partial t} = [\hat{H}, \rho(t)], \quad (1)$$

where $\hat{H} = \hat{H}_0 + \hat{V}$ is the Hamiltonian of the system, consisting of the ideal gas Hamiltonian \hat{H}_0

$$\hat{H}_0 = \frac{1}{2m} \int d^3r \frac{\partial\hat{\psi}^+(\mathbf{r})}{\partial r_k} \frac{\partial\hat{\psi}(\mathbf{r})}{\partial r_k} \quad (2)$$

and binary interaction Hamiltonian \hat{V}

$$\hat{V} = \frac{1}{2} \int d^3r \int d^3R \hat{\psi}^+(\mathbf{r} + \mathbf{R}) \hat{\psi}^+(\mathbf{r}) V(|\mathbf{R}|) \times \hat{\psi}(\mathbf{r}) \hat{\psi}(\mathbf{r} + \mathbf{R}). \quad (3)$$

Equation (1) is written in units in which Planck's constant \hbar is equal to unity. In the formulae (2), (3) m is boson mass, $V(|\mathbf{R}|)$ is the binary interaction potential, which depends only on the distance between particle, and $\hat{\psi}^+(\mathbf{r})$, $\hat{\psi}(\mathbf{r})$ are field operators. As the parameters describing the weakly non-ideal Bose gas with condensate in the kinetic stage of evolution of the system in [19], the quasiparticle distribution function $f_p(\mathbf{r}, t)$, the order parameter $\psi(\mathbf{r}, t) = |S\rangle\langle p(t)|\hat{\psi}(\mathbf{r})|S\rangle$ and superfluid velocity $v_k(\mathbf{r}, t) = m^{-1} \frac{\partial}{\partial r_k} \text{Im} \ln S\rangle\langle p(t)|\hat{\psi}(\mathbf{r})|S\rangle$ were selected.

Using the method of reduced description [23] combined with the special perturbation theory [17] allowed to obtain in [19] the following system of equations for the parameters $f_p(\mathbf{r}, t)$, $\psi(\mathbf{r}, t)$, $v_k(\mathbf{r}, t)$:

$$\begin{aligned} \frac{\partial f_p}{\partial t} + Q_p(f, \psi) &= L_p(f, \psi), \\ \frac{\partial \psi}{\partial t} + v_k \frac{\partial \psi}{\partial r_k} + \frac{1}{2} \psi \frac{\partial v_k}{\partial r_k} &= L_\psi(f, \psi), \\ \frac{\partial v_k}{\partial t} + \frac{\partial}{\partial r_k} \left\{ \frac{v^2}{2} + h(f, \psi) \right\} &= 0, \end{aligned} \quad (4)$$

where the value $L_p(f, \psi)$ is the quasiparticle collision integral and functionals $L_\psi(f, \psi)$, $Q_p(f, \psi)$, $h(f, \psi)$ are given by the expressions:

$$\begin{aligned} L_\psi(f, \psi) &= -\frac{1}{2i\psi} \int \frac{d^3p}{(2\pi)^3} \frac{\alpha_p}{\omega_p} L_p(f, \psi) + o(\lambda^5), \\ Q_p(f, \psi) &= \frac{\partial f_p}{\partial r_k} \frac{\partial}{\partial p_k} \varepsilon_p(\psi, \mathbf{v}) - \frac{\partial f_p}{\partial p_k} \frac{\partial}{\partial r_k} \varepsilon_p(\psi, \mathbf{v}) + o(\lambda), \\ h(f, \psi) &= \frac{1}{2m\psi} \left\langle \frac{\partial \omega_p}{\partial \psi} \right\rangle + \frac{\nu_0}{2} \left\langle \frac{\alpha_p}{\omega_p} \right\rangle + h_0(\psi) + o(\lambda^4), \end{aligned} \quad (5)$$

where $o(\lambda^n)$ is the value in order of magnitude λ^n and λ characterizes the smallness of the interaction between the particles, $V(|\mathbf{R}|) \sim \lambda^2$. Let us recall that the basis of Bogoliubov equilibrium state theory of Bose gas in the presence of interaction is the assumption that $\psi \sim \lambda^{-1}$, see [17]. Furthermore, it is believed that the order of magnitude of $\psi(r, t)$ does not change after differentiation $\psi(\mathbf{r}, t)$ with respect to \mathbf{r} , $(\partial\psi(\mathbf{r}, t)/\partial r_k) \sim \lambda^{-1}$. In the formulae (4) (5) the following notations were also introduced:

$$\begin{aligned} \varepsilon_p(\psi, \mathbf{v}) &\equiv \omega_p(\psi) + \mathbf{p}\mathbf{v}, \\ \langle A_p \rangle &\equiv \int d^3p f_p A_p, \quad \nu_0 \equiv \nu_{p=0}, \\ \nu_p &\equiv \int d^3R V(|\mathbf{R}|) \exp(-i\mathbf{p}\mathbf{R}), \\ h_0(\psi) &= \frac{1}{V} \sum_{p \neq 0} \frac{1}{2m} \left\{ \frac{1}{2\psi} \frac{\partial \omega_p}{\partial \psi} + \nu_0 \frac{\alpha_p - \omega_p}{\omega_p} - \nu_p \right\} \\ &\quad + \frac{\nu_0 \psi^2}{m}, \end{aligned} \quad (6)$$

where A_p is an arbitrary function of momentum p . Appearing in (5) and (6) values are given by formulae:

$$\begin{aligned} \omega_p(\psi) &\equiv [\alpha_p^2(\psi) - \beta_p^2(\psi)]^{1/2}, \\ \alpha_p(\psi) &\equiv \varepsilon_p + \beta_p(\psi), \\ \varepsilon_p &\equiv \frac{p^2}{2m}, \quad \beta_p(\psi) \equiv \frac{\nu_p \psi^2}{m}. \end{aligned} \quad (7)$$

We note that quantity ω_p represents Bogoliubov spectrum of elementary excitations.

The explicit form of the quasiparticle collision integral is not provided in the present paper because it is not necessary. As mentioned earlier, we will explore the mechanism of collisionless sound damping in a gas with BEC (i.e. Landau mechanism). The explicit form of the quasiparticle collision integral can be found in [18, 19]. Here we note only the fact that the quasiparticle collision integral $L_p(f, \psi)$ (and hence $L_\psi(f, \psi)$, see (5)) vanishes by substituting stationary Bose distribution function f_p^0

$$\begin{aligned} L_p(f^0, \psi) &= 0, \quad L_\psi(f^0, \psi) = 0, \\ f_p^0 &= \left(\exp \frac{\omega_p - \mathbf{p}\mathbf{v}}{T} - 1 \right)^{-1}, \end{aligned} \quad (8)$$

with the chemical potential of quasiparticles is equal to zero; here T is the temperature in energy units. The vanishing of the chemical potential reflects the fact that the number of the quasiparticles is not conserved during collisions [18, 21].

In the collisionless approximation equations (4) - (7) can be written as

$$\begin{aligned} \frac{\partial f_p}{\partial t} + \frac{\partial f_p}{\partial r_k} \frac{\partial}{\partial p_k} \varepsilon_p(n, \mathbf{v}) - \frac{\partial f_p}{\partial p_k} \frac{\partial}{\partial r_k} \varepsilon_p(n, \mathbf{v}) &= 0, \\ \frac{\partial n}{\partial t} + \frac{\partial}{\partial r_k} (v_k n) &= 0, \\ \frac{\partial v_k}{\partial t} + \frac{\partial}{\partial r_k} \left\{ \frac{v^2}{2} + h(f, n) \right\} &= 0, \end{aligned} \quad (9)$$

where we have introduced the new variable $n(\mathbf{r}, t)$ that is the condensate density [21]

$$\psi^2(\mathbf{r}, t) \equiv n(\mathbf{r}, t), \quad (10)$$

and the values $\varepsilon_p(n, \mathbf{v})$ and $h(f, n)$ are given by expressions

$$\begin{aligned} \varepsilon_p(n, \mathbf{v}) &\equiv \omega_p(n) + \mathbf{p}\mathbf{v}, \\ h(f, n) &= \frac{1}{m} \left\langle \nu_p \frac{\varepsilon_p}{\omega_p} \right\rangle + \frac{\nu_0}{2m} \left\langle \frac{\alpha_p}{\omega_p} \right\rangle + h_0(n), \\ h_0(n) &= \frac{1}{V} \sum_{p \neq 0} \frac{1}{2m} \left\{ \nu_p \frac{\varepsilon_p - \omega_p}{\omega_p} + \nu_0 \frac{\alpha_p - \omega_p}{\omega_p} \right\} + \frac{\nu_0 n}{m}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \omega_p(n) &\equiv [\varepsilon_p(\varepsilon_p + 2\beta_p(n))]^{1/2}, \\ \alpha_p(n) &\equiv \varepsilon_p + \beta_p(n), \quad \beta_p(n) \equiv \frac{\nu_p n}{m}, \end{aligned} \quad (12)$$

and the value ε_p is still defined by (7). Equations (9) together with (8) and (11) (12) and are the initial system of equations to study the propagation and damping of sound in dilute gas with condensate neglecting the collisions of the quasiparticles.

III. SOUND DISPERSION EQUATIONS IN DILUTED GAS WITH BEC

In order to investigate the propagation of the sound in gas with BEC, we linearize the system of equations (9),

(11) and (12) with respect to a spatially homogeneous equilibrium state according to following formulae

$$\begin{aligned} n(\mathbf{r}, t) &= n_0 + \tilde{n}(\mathbf{r}, t), \\ \mathbf{v}(\mathbf{r}, t) &= \tilde{\mathbf{v}}(\mathbf{r}, t), \\ f_{\mathbf{p}}(\mathbf{r}, t) &= f_{\mathbf{p}}^0 + \tilde{f}_{\mathbf{p}}(\mathbf{r}, t), \end{aligned} \quad (13)$$

$$n_0 \gg |\tilde{n}(\mathbf{r}, t)|, \quad f_{\mathbf{p}}^0 \gg |\tilde{f}_{\mathbf{p}}(\mathbf{r}, t)|,$$

where $n_0 = n_0(T)$ is the equilibrium value of the condensate density in the system at the temperature T . The equilibrium value of the velocity \mathbf{v}_0 is considered to be equal zero in the second formula in (13). Thus the velocity $|\tilde{\mathbf{v}}(\mathbf{r}, t)|$ is supposed to be in order of magnitude of $\tilde{n}(\mathbf{r}, t)$ and $\tilde{f}_{\mathbf{p}}(\mathbf{r}, t)$. Then the equilibrium distribution function of quasiparticles pursuant to (8) is given by expression

$$f_{\mathbf{p}}^0 = \left(\exp \frac{\omega_{\mathbf{p}}}{T} - 1 \right)^{-1}. \quad (14)$$

In this formula as in all subsequent expressions we omit the index "0" in the designation of the equilibrium value of $\omega_{\mathbf{p}}^0 \equiv \omega_{\mathbf{p}}(n_0)$ to avoid encumbering of computations. Note that the quantity $h(f^0, n_0)$ defined by (11) represents the chemical potential of atomic Bose gas $\mu = h(f^0, n_0)$ (see in this regard [18, 19]).

Deviations of the corresponding quantities from their equilibrium values were denoted by $\tilde{n}(\mathbf{r}, t)$, $\tilde{\mathbf{v}}(\mathbf{r}, t)$ and $\tilde{f}_{\mathbf{p}}(\mathbf{r}, t)$.

The equations of motion for these variables can be represented as follows:

$$\begin{aligned} \frac{\partial}{\partial t} \tilde{n}(\mathbf{r}, t) + n_0 \frac{\partial}{\partial r_k} \tilde{v}_k(\mathbf{r}, t) &= 0, \\ \frac{\partial}{\partial t} \tilde{v}_k(\mathbf{r}, t) + \frac{\nu_0}{m} \frac{\partial}{\partial r_k} \tilde{n}(\mathbf{r}, t) + \int d^3 p K_p \frac{\partial}{\partial r_k} \tilde{f}_{\mathbf{p}}(\mathbf{r}, t) &= 0, \\ \frac{\partial}{\partial t} \tilde{f}_{\mathbf{p}}(\mathbf{r}, t) + \frac{\partial \omega_{\mathbf{p}}}{\partial p_k} \frac{\partial}{\partial r_k} \tilde{f}_{\mathbf{p}}(\mathbf{r}, t) \\ - \frac{\partial f_{\mathbf{p}}^0}{\partial p_k} \frac{\partial}{\partial r_k} \left\{ (\mathbf{p} \tilde{\mathbf{v}}(\mathbf{r}, t)) + \frac{\nu_{\mathbf{p}} \varepsilon_{\mathbf{p}}}{\omega_{\mathbf{p}}} \tilde{n}(\mathbf{r}, t) \right\} &= 0, \end{aligned} \quad (15)$$

where we have introduced the following notation

$$K_p = \frac{1}{2m(2\pi)^3} \frac{\varepsilon_p(\nu_0 + 2\nu_p) + \nu_p \nu_0 n_0}{\omega_p}. \quad (16)$$

Recall that in the present paper Planck constant \hbar is considered be equal to unity. Note also that deriving equations (15) we have discarded terms that are higher than the first order of magnitude λ . It was taken into account the circumstance of (10) that $n_0 \sim \lambda^{-2}$ or i.e. $\nu_p n_0 \sim \lambda^0$.

Proceeding further to the Fourier transform of quantities $\tilde{n}(\mathbf{r}, t)$, $\tilde{\mathbf{v}}(\mathbf{r}, t)$, $\tilde{f}_{\mathbf{p}}(\mathbf{r}, t)$ in equations (15) as provided by formula

$$\tilde{\zeta}(\mathbf{r}, t) = \int_{-\infty}^{\infty} d\omega \int d^3 q e^{i\mathbf{q}\mathbf{r} - i\omega t} \zeta(\mathbf{q}, \omega), \quad (17)$$

where $\tilde{\zeta}(\mathbf{r}, t)$ should be understood as either of the considered quantities, we obtain

$$\begin{aligned} \omega n(\mathbf{q}, \omega) &= n_0 \mathbf{q} \mathbf{v}(\mathbf{q}, \omega), \\ \omega v_k(\mathbf{q}, \omega) &= \frac{\nu_0}{m} q_k n(\mathbf{q}, \omega) + q_k A(\mathbf{q}, \omega), \\ \left(\omega - q_k \frac{\partial \omega_{\mathbf{p}}}{\partial p_k} \right) f_{\mathbf{p}}(\mathbf{q}, \omega) &= \\ -q_k \frac{\partial f_{\mathbf{p}}^0}{\partial p_k} \left[\mathbf{p} \mathbf{v}(\mathbf{q}, \omega) + \frac{\nu_{\mathbf{p}} \varepsilon_{\mathbf{p}}}{\omega_{\mathbf{p}}} n(\mathbf{q}, \omega) \right], \end{aligned} \quad (18)$$

where we denote (see (15), (16))

$$\begin{aligned} A(\mathbf{q}, \omega) &\equiv \int d^3 p f_{\mathbf{p}}(\mathbf{q}, \omega) K_p \\ &= \frac{1}{2m(2\pi)^3} \int d^3 p f_{\mathbf{p}}(\mathbf{q}, \omega) \frac{\varepsilon_p(\nu_0 + 2\nu_p) + \nu_p \nu_0 n_0}{\omega_p}. \end{aligned} \quad (19)$$

In the formulae (18) (19) in the Fourier transforms of the quantities $\tilde{n}(\mathbf{r}, t)$, $\tilde{\mathbf{v}}(\mathbf{r}, t)$, $\tilde{f}_{\mathbf{p}}(\mathbf{r}, t)$ we omit the 'tilde' sign.

Further expressing the values $n(\mathbf{q}, \omega)$ and $\mathbf{v}(\mathbf{q}, \omega)$ from the first two equations of (18) in terms of $A(\mathbf{q}, \omega)$,

$$\begin{aligned} n(\mathbf{q}, \omega) &= \frac{n_0 q^2}{\omega^2 - u_0^2 q^2} A(\mathbf{q}, \omega), \\ v_k(\mathbf{q}, \omega) &= \frac{\omega q_k}{\omega^2 - u_0^2 q^2} A(\mathbf{q}, \omega), \end{aligned} \quad (20)$$

the third equation of (18) can be written in the form

$$\begin{aligned} \left(\omega - q_k \frac{\partial \omega_{\mathbf{p}}}{\partial p_k} \right) f_{\mathbf{p}}(\mathbf{q}, \omega) &= -q_k \frac{\partial \omega_{\mathbf{p}}}{\partial p_k} \frac{\partial f_{\mathbf{p}}^0}{\partial \omega_p} \\ &\times \frac{\mathbf{p} \mathbf{q} \omega \omega_{\mathbf{p}} + \nu_{\mathbf{p}} \varepsilon_{\mathbf{p}} n_0 q^2}{\omega_p (\omega^2 - u_0^2 q^2)} A(\mathbf{q}, \omega), \end{aligned} \quad (21)$$

where u_0 is so-called speed of zero sound in Bose gas

$$u_0^2 = \frac{\nu_0 n_0}{m}. \quad (22)$$

As is readily seen, equation (21) subject to (19) is an integral equation for distribution function Fourier transform $f_{\mathbf{p}}(\mathbf{q}, \omega)$. The solution of this equation can be represented as in [25]:

$$f_p(\mathbf{q}, \omega) = B_p(\mathbf{q}) \delta(\omega - \mathbf{q}\mathbf{u}_p) - \mathbf{q}\mathbf{u}_p \frac{\partial f_p^0}{\partial \omega_p} \frac{\mathbf{p}\mathbf{q}\omega\omega_p + \nu_p \varepsilon_p n_0 q^2}{\omega_p (\omega^2 - u_0^2 q^2) (\omega - \mathbf{q}\mathbf{u}_p + i0)} A(\mathbf{q}, \omega), \quad (23)$$

where we have introduced the following notation:

$$\mathbf{u}_p \equiv \frac{\partial \omega_p}{\partial \mathbf{p}}, \quad (24)$$

and $B_p(\mathbf{q})$ is an arbitrary function, which is required to impose the following restriction: the distribution function $\tilde{f}_p(\mathbf{r}, t)$, calculated in accordance with (13), (17) and (23) should be small compared with the equilibrium distribution function f_p^0 .

It implies also from (17) that $B_p(\mathbf{q})$ must satisfy the following relation:

$$B_p^*(\mathbf{q}) = B_p(-\mathbf{q}). \quad (25)$$

All the valid set of such functions we denote by $B_{\sigma p}(\mathbf{q})$, where σ is a continuous or discrete symbolic parameter such that the functions $B_p(\mathbf{q}) \equiv B_{\sigma p}(\mathbf{q})$ may depend on σ . The reason for introducing this index may consist, for example, the following: the set of functions $B_{\sigma p}(\mathbf{q})$ should be enough to build an arbitrary value for the distribution function $\tilde{f}_p(\mathbf{r}, t)$ at the initial time $t = 0$ (see in this context also [26]).

Formula (23) allows to find the value $A(\mathbf{q}, \omega)$ in terms of $B_{\sigma p}(\mathbf{q})$ functions:

$$A_\sigma(\mathbf{q}, \omega) = B_\sigma(\mathbf{q}, \omega) \varepsilon^{-1}(\mathbf{q}, \omega), \quad (26)$$

where

$$B_\sigma(\mathbf{q}, \omega) = \frac{1}{2m(2\pi)^3} \int d^3p \frac{\nu_0 n_0 \nu_p + \varepsilon_p (\nu_0 + 2\nu_p)}{\omega_p} \times B_{\sigma p}(\mathbf{q}) \delta(\omega - \mathbf{q}\mathbf{u}_p) \quad (27)$$

and

$$\varepsilon(\mathbf{q}, \omega) \equiv 1 + \frac{1}{2m(2\pi)^3} \int d^3p \frac{\partial f_p^0}{\partial \omega_p} (\mathbf{q}\mathbf{p}\omega\omega_p + \nu_p \varepsilon_p n_0 q^2) \times \frac{\mathbf{q}\mathbf{u}_p [\nu_0 n_0 \nu_p + \varepsilon_p (\nu_0 + 2\nu_p)]}{\omega_p^2 (\omega^2 - u_0^2 q^2) (\omega - \mathbf{q}\mathbf{u}_p + i0)}. \quad (28)$$

The expression (23) for given values $f_p(\mathbf{q}, \omega)$ subject to (26) - (28) can now be represented in the form:

$$f_{\sigma p}(\mathbf{q}, \omega) = B_{\sigma p}(\mathbf{q}) \delta(\omega - \mathbf{q}\mathbf{u}_p) + \varepsilon^{-1}(\mathbf{q}, \omega) \mathbf{q}\mathbf{u}_p \frac{\partial f_p^0}{\partial \omega_p} \times \frac{\mathbf{p}\mathbf{q}\omega\omega_p + \nu_p \varepsilon_p n_0 q^2}{\omega_p (\omega^2 - u_0^2 q^2) (\omega - \mathbf{q}\mathbf{u}_p + i0)} B_\sigma(\mathbf{q}, \omega). \quad (29)$$

We remark that in case of charged particles gas the value $\varepsilon(\mathbf{q}, \omega)$ (see (28))

$$\varepsilon(\mathbf{q}, \omega) = \varepsilon_1(\mathbf{q}, \omega) + i\varepsilon_2(\mathbf{q}, \omega) \quad (30)$$

represents complex dielectric permittivity of the system (see e.g. [23]). It is known that the presence of imaginary term in dielectric permittivity indicates the energy dissipation of electromagnetic waves with dispersion relation that should be obtained from the equation

$$\varepsilon(\mathbf{q}, \omega_0(\mathbf{q}) - i\gamma_q) = 0. \quad (31)$$

Moreover, the wave decrement γ_q is determined by the imaginary part $\varepsilon_2(\mathbf{q}, \omega)$ of value $\varepsilon(\mathbf{q}, \omega)$. For this reason, weakly damped oscillations in the system

$$|\omega_0(\mathbf{q})| \gg \gamma_q \quad (32)$$

can exist if only

$$|\varepsilon_1(\mathbf{q}, \omega)| \gg |\varepsilon_2(\mathbf{q}, \omega)|, \quad (33)$$

besides, as the consequence of (30) - (33) (see in this regard [25, 26]), the frequency $\omega_0(\mathbf{q})$ can be found from the equation

$$\varepsilon_1(\mathbf{q}, \omega_0(\mathbf{q})) = 0 \quad (34)$$

and the damping rate γ_q is given by expression

$$\gamma_q = \left\{ \frac{\partial \varepsilon_1(\mathbf{q}, \omega)}{\partial \omega} \right\}_{\omega=\omega_0}^{-1} \varepsilon_2(\mathbf{q}, \omega_0(\mathbf{q})). \quad (35)$$

The structure of solution (29) is such that weakly damped waves may also be excited in investigated system, in accordance with (20) and (17), and the dispersion law is determined by (31) - (33). As we will show in the following section, such waves would represent sound waves in weakly nonideal Bose gas with condensate. The value γ_q obtained according to (29) - (32) will determine the attenuation of sound in investigated system.

IV. THE DAMPING RATE OF SOUND IN DILUTE BOSE GAS WITH CONDENSATE

In order to solve this problem it is necessary to determine the real $\varepsilon_1(\mathbf{q}, \omega)$ and imaginary $\varepsilon_2(\mathbf{q}, \omega)$ parts of the value $\varepsilon(\mathbf{q}, \omega)$. For this purpose in the expression (28) we use the formula

$$\frac{1}{x+i0} = P\frac{1}{x} - i\pi\delta(x), \quad (36)$$

where the symbol P means that the further integration is taken in means of Cauchy principal value. After some hackneyed transformations we obtain the following expressions for $\varepsilon_1(\mathbf{q}, \omega)$ and $\varepsilon_2(\mathbf{q}, \omega)$:

$$\begin{aligned} \varepsilon_1(\mathbf{q}, \omega) &= 1 + \frac{1}{2m(2\pi)^3} P \int d^3p \frac{\partial f_p^0}{\partial \omega_p} \\ &\times \frac{\mathbf{q}\mathbf{u}_p (\mathbf{q}\mathbf{p}\omega\omega_p + \nu_p \varepsilon_p n_0 q^2) [\nu_0 n_0 \nu_p + \varepsilon_p (\nu_0 + 2\nu_p)]}{\omega_p^2 (\omega^2 - u_0^2 q^2) (\omega - \mathbf{q}\mathbf{u}_p)}, \\ \varepsilon_2(\mathbf{q}, \omega) &= -\frac{\pi\omega}{2m(2\pi)^3} \int d^3p \frac{\partial f_p^0}{\partial \omega_p} (\mathbf{q}\mathbf{p}\omega\omega_p + \nu_p \varepsilon_p n_0 q^2) \\ &\times \frac{[\nu_0 n_0 \nu_p + \varepsilon_p (\nu_0 + 2\nu_p)]}{\omega_p^2 (\omega^2 - u_0^2 q^2)} \delta(\omega - \mathbf{q}\mathbf{u}_p). \end{aligned} \quad (37)$$

Further calculations is not possible without specifying the explicit form of ν_p that is the Fourier transform of the interaction potential $V(|\mathbf{R}|)$ (see (3), (6)). To simplify the calculations, in many cases it is accepted to replace the interaction potential $V(|\mathbf{R}|)$ by the following effective potential (see, in this regard [6, 7])

$$V(|\mathbf{R}|) = \nu_0 \delta(\mathbf{R}), \quad \nu_0 = \frac{4\pi a_{sc}}{m}, \quad (38)$$

where a_{sc} is so-called scattering length (for details see [6]). Thus we have $\nu_p \equiv \nu_0$. Taking this into account we have $\omega_p \equiv \omega_p$ and the value \mathbf{u}_p (see (1.12) (24)) can be expressed in the form

$$\mathbf{u}_p = \frac{\mathbf{p}}{p} u_p, \quad u_p \equiv \frac{\partial \omega_p}{\partial p} = \frac{\varepsilon_p + \nu_0 n_0}{\omega_p} \frac{p}{m}. \quad (39)$$

Formulae (38) and (39) enable us to perform integration in (37) over the angle between vectors \mathbf{q} and \mathbf{p} , and then $\varepsilon_1(\mathbf{q}, \omega)$ can be represented in the form

$$\begin{aligned} \varepsilon_1(q, \omega) &= 1 - \frac{\nu_0}{4\pi^2 (\omega^2 - u_0^2 q^2)} \int_0^\infty \frac{dp p^2}{\omega_p^2} \frac{\partial f_p^0}{\partial \omega_p} \\ &\times \left[\frac{p}{m u_p} \omega^2 \omega_p + \varepsilon_p u_0^2 q^2 \right] (n_0 \nu_0 + 3\varepsilon_p) \\ &\times \left\{ 1 - \frac{\omega}{2q u_p} \ln \left| \frac{\omega + q u_p}{\omega - q u_p} \right| \right\}. \end{aligned} \quad (40)$$

After the same angle integration the second of expressions (37) can be written as follows

$$\begin{aligned} \varepsilon_2(q, \omega) &= -\frac{\nu_0 \omega}{8\pi m (\omega^2 - u_0^2 q^2)} \int_0^\infty \frac{dp p^2}{\omega_p^2 u_p} \frac{\partial f_p^0}{\partial \omega_p} \\ &\times [(p/u_p) \omega^2 \omega_p + \varepsilon_p \nu_0 n_0 q^2] (n_0 \nu_0 + 3\varepsilon_p) \theta(q u_p - |\omega|), \end{aligned} \quad (41)$$

where $\theta(x)$ is Heaviside step function.

We now define, following the work [12], the sound damping rate in the case of low ($T \ll \nu_0 n_0$) and high ($T \gg \nu_0 n_0$ but $T < T_c$) temperatures. To study the case of low temperatures it is convenient to use variable $z \equiv \omega_p/T$ in the integration in expressions (40) and (41). The variables p , ε_p , u_p in terms of the variable z can be expressed using the ω_p explicit expression (12) and also (7) and (39). Carrying out all rather cumbersome but necessary calculations, the result can be summarized as follows:

$$\begin{aligned} \varepsilon_1(q, \omega) &= 1 - \frac{F}{a (\omega^2 - u_0^2 q^2)} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} g(q, \omega, z) \\ &\times \left\{ 1 - \frac{\omega}{2q u_0 v(z)} \ln \left| \frac{\omega + q u_0 v(z)}{\omega - q u_0 v(z)} \right| \right\}, \\ \varepsilon_2(q, \omega) &= -\frac{\pi F}{2a (\omega^2 - u_0^2 q^2)} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} g(q, \omega, z) \\ &\times \frac{\omega}{q u_0 v(z)} \theta(q u_0 v(z) - |\omega|), \quad f^0(z) = \frac{1}{e^z - 1}, \end{aligned} \quad (42)$$

where the functions $g(q, \omega, z)$ and $v(z)$ are given by:

$$\begin{aligned} g(q, \omega, z) &\equiv \frac{\sqrt{2} \sqrt{1 + a^2 z^2} - 1 (3\sqrt{1 + a^2 z^2} - 2)}{z (1 + a^2 z^2)} \\ &\times \left[q^2 u_0^2 (1 + a^2 z^2 - \sqrt{1 + a^2 z^2}) + \omega^2 a^2 z^2 \right], \\ v(z) &\equiv \frac{\sqrt{2} \sqrt{1 + a^2 z^2} \sqrt{\sqrt{1 + a^2 z^2} - 1}}{a z} \end{aligned} \quad (43)$$

and the following notations were introduced (recall u_0 still given by (22)):

$$\begin{aligned} a &\equiv \frac{T}{\nu_0 n_0}, \\ F &\equiv \frac{1}{4\pi^2} \nu_0 m^2 u_0 = \frac{2}{\sqrt{\pi}} \sqrt{n_0 a_{sc}^3}. \end{aligned} \quad (44)$$

Deriving the last formula of (44) it is necessary to use expressions (22) and (38). The quantity $n_0 a_{sc}^3$ is called the gas parameter, see [6, 7]. That is, in diluted gases the following relation should be satisfied:

$$n_0 a_{sc}^3 \ll 1. \quad (45)$$

It is easy to verify that the values $\varepsilon_1(q, \omega)$ and $\varepsilon_2(q, \omega)$ can be represented as functions of one dimensionless variable $s \equiv \omega(q)/u_0 q$

$$\begin{aligned} \varepsilon_1(s) &= 1 - \frac{F}{a(s^2 - 1)} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} g(s, z) \\ &\times \left\{ 1 - \frac{s}{2v(z, a)} \ln \left| \frac{s + v(z, a)}{s - v(z, a)} \right| \right\}, \\ \varepsilon_2(s) &= -\frac{\pi F s}{2a(s^2 - 1)} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} \frac{g(s, z)}{v(z, a)} \\ &\times \theta(v(z, a) - |s|), \end{aligned} \quad (46)$$

where

$$\begin{aligned} g(s, z) &\equiv \frac{\sqrt{2}\sqrt{\sqrt{1+a^2z^2}-1}(3\sqrt{1+a^2z^2}-2)}{z(1+a^2z^2)} \\ &\times \left[1 + a^2z^2 - \sqrt{1+a^2z^2} + s^2a^2z^2 \right], \end{aligned} \quad (47)$$

and for these functions the symmetry conditions are valid

$$\varepsilon_1(s) = \varepsilon_1(-s), \quad \varepsilon_2(s) = \varepsilon_2(-s). \quad (48)$$

Expressions (34) and (35) that determine the dispersion and decrement (or increment) of small oscillations in investigated system, can be written in the following form by taking into account (46):

$$\begin{aligned} \varepsilon_1(s_0) &= 0, \quad s_0 \equiv \frac{\omega_0(q)}{u_0 q}, \\ \gamma_q &= u_0 q \left\{ \varepsilon_2(s) \left(\frac{\partial \varepsilon_1(s)}{\partial s} \right)^{-1} \right\}_{s=s_0}. \end{aligned} \quad (49)$$

As is easily seen, these oscillations have linear spectrum. In virtue of (46) and (49) the structure of the dispersion equation is such that the unknown value s , which determines the oscillation frequency as a function of the wave vector, does not depend on wave vector itself.

In accordance with (34) and (49) the dependence of the oscillation frequency ω_0 on the wave vector q in this system should be determined by the solution of equation

$$\begin{aligned} s_0^2 - 1 &= \frac{F}{a} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} g(s_0, z) \\ &\times \left\{ 1 - \frac{s_0}{2v(z)} \ln \left| \frac{s_0 + v(z)}{s_0 - v(z)} \right| \right\}. \end{aligned} \quad (50)$$

We note that dispersion equation (50) is similar to the dispersion equation of zero sound in a normal Fermi liquid, (compare e.g. with the corresponding formula in

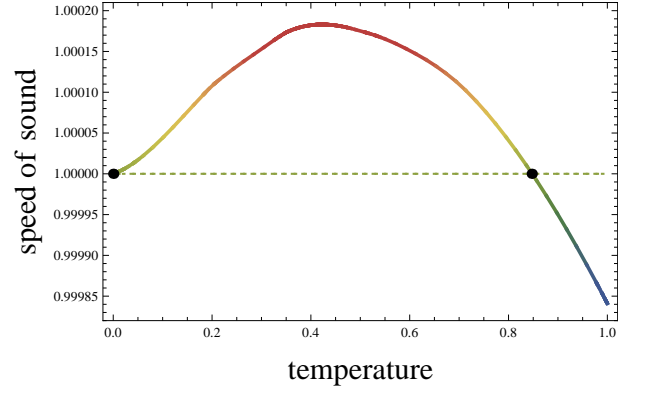


FIG. 1: Dependence of dimensionless speed of sound $s(a) \equiv \omega_0(q, a)/u_0 q$ on temperature (dimensionless quantity a , see (44)) obtained by a numerical solution of (50) for $F = 10^{-3}$ (solid line). Dots display intersection with line $s = 1$ (dashed one) in points $a = 0$ and $a \approx 0.847$.

[25]). Further, taking this equation into account to calculate the derivative $\frac{\partial \varepsilon_1(s)}{\partial s}$ appearing in (49), the damping rate γ_q can be presented as follows:

$$\frac{\gamma_q}{u_0 q} = -\frac{\pi}{2} \frac{F s_0}{a b(s_0)} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} \frac{g(s_0, z)}{v(z)} \theta(v(z) - |s_0|), \quad (51)$$

where

$$\begin{aligned} b(s_0) &\equiv \left\{ (s^2 - 1) \frac{\partial \varepsilon_1(s)}{\partial s} \right\}_{s=s_0} \\ &= \frac{s_0^2 + 1}{s_0} + \frac{F}{a} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} \left\{ \frac{g(s_0, z)}{s_0} - \frac{\partial g(s_0, z)}{\partial s_0} \right\} \\ &+ \frac{F s_0}{2a} \left\{ \frac{\partial}{\partial s_0} \int_0^\infty dz \frac{\partial f^0(z)}{\partial z} \frac{g(s_0, z)}{v(z)} \ln \left| \frac{s_0 + v(z)}{s_0 - v(z)} \right| \right\}. \end{aligned} \quad (52)$$

We emphasize that mentioned above expressions (42) - (52) are exact, despite the fact that we have modified them to the form suitable to study the low temperature regime.

As is readily seen, equation (50) in the general case can be solved only by numerical methods. Figure 1 shows the dependence $s(a)$ (see (46) and (44)) obtained as a result of numerical solution of equation (50) for $F = 10^{-3}$. It is evident that the function $s(a)$ behaves nonmonotonically as a changes. At low a region (the case of low temperature) increase of function is observed. At the point $a \approx 0.43$ it reaches maximum, and then $s(a)$ decreases monotonically as a increases, and in the point $a \approx 0.847$ $s(a)$ equals unity again. As mentioned above, the increase of value a is restricted at least the critical temperature, see (44). At zero temperature ($a = 0$) we deal with a classical zero sound in Bose system, because $s = 1$, and hence $\omega_0(q) = u_0 q$ due to (46). This result is naturally

expected. However, we note one more time that at the point $a \approx 0.847$ also holds $s = 1$ and the frequency of sound in dilute gas with Bose condensate equals again to the frequency of zero sound in such a system. The fact that we discovered is still not mentioned in the literature. It may be a consequence of use of the evolution equations of the system in present work, that were obtained in the microscopic approach, as well as the numerical solution of the dispersion equations. Whatever the case is, the very existence of the second point of a sound dispersion curve with $s = 1$, that is $a \approx 0.847$ (or $T \approx 0.847 \nu_0 n_0$) for $F = 10^{-3}$, requires an individual physical interpretation. Some more comments regarding this point will be given below.

In the regions of low ($T \ll \nu_0 n_0$) and high ($T \gg \nu_0 n_0$ but $T < T_c$) temperatures a solution of (50) can be expressed in analytical form. Thus one can obtain analytical expressions for the quantity γ_q in two limiting cases. To do this, as we shall see, one need to involve a numerical analysis as an auxiliary technique.

Consider first the case of low temperatures. By virtue of inequality (low temperature regime, as mentioned above)

$$a = \frac{T}{\nu_0 n_0} \ll 1, \quad (53)$$

and the rapid decrease of function $f^0(z)$ as $z \rightarrow \infty$ (see (42)), functions $g(q, \omega, z)$ and $u(z)$ can be expanded in powers series of az in (46) - (48):

$$g(s, z) \approx a^3 z^2 \left(s^2 + \frac{1}{2} \right), \quad v(z) \approx 1 + \frac{3}{8} a^2 z^2. \quad (54)$$

As will become apparent from the subsequent formulae such expansion corresponds to the development of perturbation theory with respect to small parameter a . Taking into account expansions (54) the dispersion equation (50) can be written as:

$$\begin{aligned} s_0^2 - 1 &= F a^2 \left(s_0^2 + \frac{1}{2} \right) \int_0^\infty dz z^2 \frac{\partial f^0(z)}{\partial z} \\ &\times \left\{ 1 - \frac{s_0}{2v(z)} \ln \left| \frac{s_0 + 1 + \frac{3}{8} a^2 z^2}{s_0 - 1 - \frac{3}{8} a^2 z^2} \right| \right\}, \\ s_0 &\equiv \frac{\omega_0(q)}{u_0 q}. \end{aligned} \quad (55)$$

In the region of small a , i.e. low temperatures, as we have seen from the Figure 1, $s \approx 1$. That is to say that value $\delta s \equiv s - 1$ should be much less than unity, $|\delta s| \ll 1$. Therefore, the equation (55) admits further simplification

$$\begin{aligned} \delta s &\approx \frac{3}{4} F a^2 \int_0^\infty dz z^2 \frac{\partial f^0(z)}{\partial z} \\ &\times \left\{ 1 - \frac{1}{2} \ln 2 + \frac{1}{2} \ln \left| \delta s - \frac{3}{8} a^2 z^2 \right| \right\}. \end{aligned} \quad (56)$$

When $F \ll 1$ and $a \ll 1$ (see (44), (49)) the solution of (56) can exist only in the $\delta s \ll a^2 \ll 1$ domain. This inequality makes it possible to neglect the value δs inside logarithm term in the integrand in (56) as the first approximation of perturbation theory. As the result we obtain

$$\delta s = F a^2 d, \quad \delta s \equiv s - 1, \quad (57)$$

where the notation d is

$$d = \frac{3}{4} \int_0^\infty \frac{dz z^2 e^z}{(e^z - 1)^2} \left\{ \frac{1}{2} \ln 2 - 1 - \frac{1}{2} \ln \frac{3}{8} a^2 z^2 \right\}. \quad (58)$$

Formula (57) is also confirmed by numerical calculations. Fig. 2 shows the dependence $d(\delta s)$, $\delta s \equiv s - 1$, computed according to the formula (56)

$$\begin{aligned} d(\delta s) &\equiv \frac{3}{4} \int_0^\infty dz \frac{z^2 e^z}{(e^z - 1)^2} \\ &\times \left\{ \frac{1}{2} \ln 2 - 1 - \frac{1}{2} \ln \left| \delta s - \frac{3}{8} a^2 z^2 \right| \right\}. \end{aligned} \quad (59)$$

As can be seen from the plot on Fig. 2, the equality $d(\delta s) \approx d \approx 4.4$ for $a = 0.1$ holds up to the second decimal. As a consequence, in the expression for d that is given above, the dependence on δs can be neglected. Then one can derive:

$$d \approx -\frac{\pi^2}{4} \ln a - 1.27, \quad (60)$$

where the second term was calculated numerically and deriving the first term the following integral value was used:

$$\int_0^\infty \frac{dz z^2 e^z}{(e^z - 1)^2} = \frac{\pi^2}{3}.$$

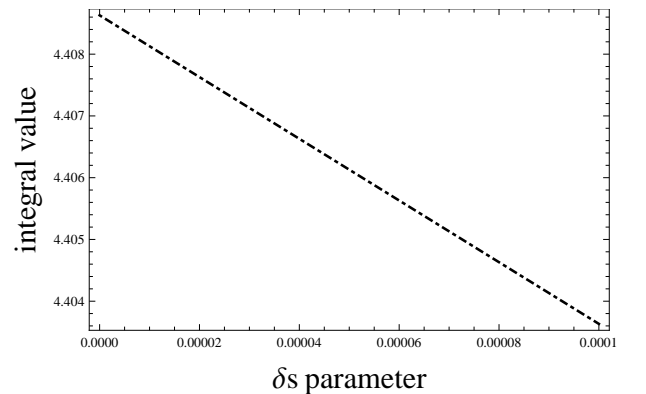


FIG. 2: Dependence $d(\delta s)$ plotted numerically according to (59) for $a = 0.1$.

Thus the solution (57) shows the following dependence of the sound frequency in dilute gas with BEC on the low temperature range:

$$\omega_0(q) \approx \pm u_0 q (1 + F a^2 d), \quad a = \frac{T}{\nu_0 n_0} \ll 1 \quad (61)$$

Analytical dependence $s(a)$ in case of $a \ll 1$ in accordance with (57) and (62) is shown on Fig.3 for three different values of F .

Damping rate of sound in BEC calculated according to formulae (51), (52) and using (54), (57) - (61) in the first order approximation with respect to a is given by a fairly simple expression

$$\frac{\gamma_q}{\omega_0(q)} \approx \frac{\gamma_q}{u_0 q} \approx \frac{\pi^3}{8} F a^2 = \frac{\pi^{5/2}}{2} \sqrt{n_0 a_{sc}^3} \left(\frac{T}{\nu_0 n_0} \right)^2. \quad (62)$$

If we assume that at low temperatures (see (53)) all Bose gas is in the condensate [12], $n_0(T) \approx n_0(0) = n_0$, the quadratic dependence of the decrement γ_q on temperature follows from (61). This behavior of sound decrement at low temperatures significantly differs from that given in [1, 12]. In these papers the value γ_q is known to depend in the low-temperature regime on the temperature as T^4 . This is obviously a consequence of our use of the coupled evolution equations derived in the microscopic approach from the first principles.

Fig. 4 shows the dimensionless damping rate $\gamma_q/\omega_0(q)$ at low temperatures (low values of a) plotted for $F = 10^{-3}$. The solid line in the figure reflects the behavior of the dimensionless decrement that follows from analytical expression (62). The dotted line shows the dependence $\gamma_q/\omega_0(q)$ on the temperature obtained as a result of numerical calculations based on formulae (50) - (52). It is evident that analytical expression (62) coincides with good accuracy the damping rate temperature dependence in a dilute gas with BEC in a rather wide range of low temperatures.

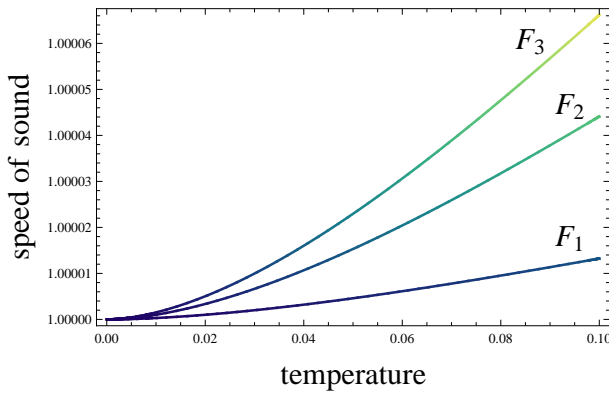


FIG. 3: Dependence $\delta s(a)$, $\delta s \equiv \omega_0(q, a)/u_0 q - 1$, plotted according to analytical expressions (57), (60) for $F_1 = 3 \cdot 10^{-4}$, $F_2 = 10^{-3}$ and $F_3 = 1.5 \cdot 10^{-3}$ respectively.

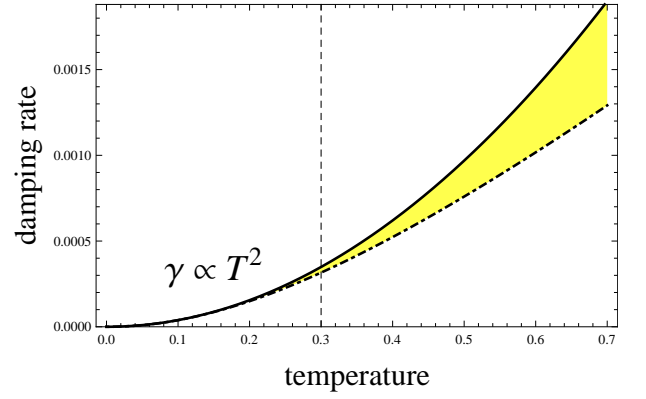


FIG. 4: Dependence of dimensionless damping rate γ_q/ω_0 on temperature (dimensionless quantity a , see (53)). Solid line represents the analytical approach given by (62) and dot-dashed line represents numerical calculation in accordance with (50) - (52). Both lines were plotted for $F = 10^{-3}$. Dashed vertical line indicated the region where analytical formula (53) works with good accuracy.

Here we should make the following remark. The experimental data (see [3, 4] and [27-31]) show that under present experimental conditions, the criterion $a = (T/\nu_0 n_0) \ll 1$ is generally not realized. For example, the approximate estimates of the parameter a obtained in accordance with typical experimental conditions of mentioned studies, show the following circumstance. This parameter takes on the value $a \approx 2$ for ^{85}Rb [30], $a \approx 23$ for ^{23}Na [4], $a \approx 58$ for ^1H [27]. The values of the scattering lengths in order to perform these estimates were taken from [6]. The exceptions are the data of [28] and [31]. For ^7Li we have $a \approx 0.5$ [28] and for ^{133}Cs accordingly $a \approx 0.8$ [31]. Therefore, to observe the effect of sound attenuation in dilute gases with BEC at low temperatures the most promising are experimental conditions in [28, 31].

Experimental conditions in other studies are rather close to the limiting case opposite to (53), that is the case of high temperatures. For this reason we now consider, as in [12], the other limiting case

$$a \equiv \frac{T}{\nu_0 n_0} \gg 1, \quad T < T_c. \quad (63)$$

As before we assume here that $\varepsilon_p \sim \nu_0 n_0$. Then from (63) follows that $\omega_p(n_0) \ll T$. In this case the following limiting expression for the distribution function $f_p^0(\omega_p)$ can be used (see, in this regard (14) and [12])

$$f_p^0(\omega_p) \approx \frac{T}{\omega_p}, \quad \varepsilon_p \sim \nu_0 n_0, \quad \omega_p(n_0) \ll T. \quad (64)$$

In order to simplify further calculations in (40) and (41) it is convenient to change integration variable pt

variable z , that is introduced in accordance with the formula

$$\frac{p^2}{2m\nu_0 n_0} = z.$$

As a result of the change of integration variable we obtain the following expressions:

$$\varepsilon_1(s) = 1 - \frac{Fa}{(s^2 - 1)} \int_0^\infty dz g(s, z) \times \left\{ 1 - \frac{s}{2v(z)} \ln \left| \frac{s + v(z)}{s - v(z)} \right| \right\}, \quad (65)$$

where now we have introduced the notations

$$g(s, z) \equiv \frac{\sqrt{2}(3z + 1) [z(s^2 + 1) + 2s^2 + 1]}{\sqrt{z}(z + 1)(z + 2)^2}, \quad (66)$$

$$v(z) \equiv \sqrt{2} \frac{z + 1}{\sqrt{z + 2}},$$

and the quantities F and a are still defined by (44) and (45).

From the condition $\varepsilon_1(s_0) = 0$ (see (2.19), (35), (61), (65)), written when $s_0(a) \approx \pm(1 + \delta s(a))$, $|\delta s| \ll 1$, in the form of

$$\delta s = Fa \int_0^\infty dz \frac{(3z + 1)(2z + 3)}{\sqrt{2z}(z + 1)(z + 2)^2} \times \left\{ 1 - \frac{1}{2v(z)} \ln \left| \frac{1 + v(z) + \delta s}{1 - v(z) + \delta s} \right| \right\}, \quad (67)$$

$$\delta s(a) \equiv s(a) - 1,$$

follows that the equation (61) has a solution only if $Fa \ll 1$ (but $a \gg 1$ see (63)). The analytical solution of this equation does not exist. However, numerical analysis reveals that the result of integration with respect to z in the right-hand side of formula (67) almost does not depend on δs if $|\delta s| \ll 1$. In this regard, the solution of (67) in the case of $|\delta s| \ll 1$ can be written as follows:

$$\delta s = -Fad_2, \quad Fa \ll 1, \quad (68)$$

where the constant d_2 is given by

$$d_2 \equiv \int_0^\infty dz \frac{(3z + 1)(2z + 3)}{\sqrt{2z}(z + 1)(z + 2)^2} \times \left\{ 1 - \frac{\sqrt{z + 2}}{2\sqrt{2}(z + 1)} \ln \left| \frac{\sqrt{z + 2} + \sqrt{2}(z + 1)}{\sqrt{z + 2} - \sqrt{2}(z + 1)} \right| \right\} \approx 3.97. \quad (69)$$

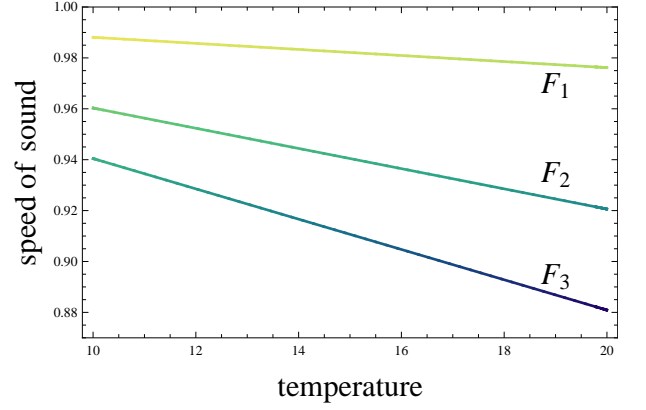


FIG. 5: Analytical dependence of dimensionless speed of sound $s \equiv \omega_0(q, a)/u_0 q$ on temperature (dimensionless quantity a) in case of "high" temperatures, plotted according to expressions (68), (69) for $F_1 = 3 \cdot 10^{-4}$, $F_2 = 10^{-3}$ and $F_3 = 1.5 \cdot 10^{-3}$ respectively.

Analytical dependence $s(a)$ in case of $a \gg 1$ in accordance with (68) and (69) is shown on Fig.4 for three different values of F .

The value $b(s)$ required in accordance with (51), (52) to calculate the damping rate of sound at high temperatures (see (63)) is given by:

$$b(s) \equiv \left\{ (s^2 - 1) \frac{\partial \varepsilon_1(s)}{\partial s} \right\}_{\omega=\omega_0} = \frac{s_0^2(a) + 1}{s_0(a)} - \frac{Fa}{s_0(a)} \int_0^\infty dz g(s_0(a), z) \frac{v^2(z)}{s_0^2(a) - v^2(z, a)} - \frac{Fa}{s_0(a)} \int_0^\infty dz \frac{\partial g(s_0(a), z)}{\partial s_0(a)} \times \left\{ 1 - \frac{s_0(a)}{2v(z, a)} \ln \left| \frac{s_0(a) + v(z, a)}{s_0(a) - v(z, a)} \right| \right\}, \quad (70)$$

where the functions $g(s, z)$ and $v(z)$ are still defined by (66), and $s_0(a)$ is given by (67) taking into account (68). One can verify that the main contribution to $b(s)$ the first term only yields.

The damping rate γ_q of sound in BEC, calculated according to formulae (51) using (70), in the main approximation with respect to Fa (see (68)) indicates a linear dependence on temperature:

$$\frac{\gamma_q}{\omega_0(q)} \approx \frac{\gamma_q}{u_0 q} \approx \frac{\pi(\pi + 6)}{8} Fa, \quad (71)$$

and that is a well known result (see for example [2, 12]). Deriving (71) the following integral was used

$$\int_0^\infty dz \frac{(3z + 1)(2z + 3)}{\sqrt{z}(z + 1)^2(z + 2)^{3/2}} = \frac{\pi + 6}{2}.$$

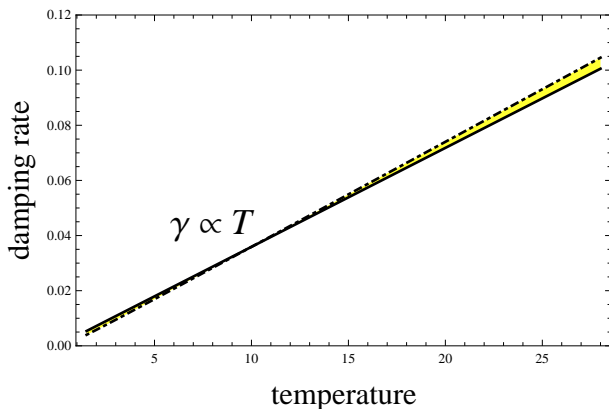


FIG. 6: Dependence of dimensionless damping rate γ_q/ω_0 on the temperature (dimensionless quantity a) in the case of 'high' temperatures. The solid line shows the dependence (71) and the dashed one reveals the result of numerical computing based on the formulae (50) - (52). Both lines were plotted for $F = 10^{-3}$.

Formula (71) using (22), (38), (44) can be written as

$$\gamma_q = \frac{\pi + 6}{8} a_{sc} T q. \quad (72)$$

It should be mentioned that this expression differs *prima facie* from that given in [2, 12], where

$$\gamma_q = \frac{3\pi}{8} a_{sc} T q. \quad (73)$$

However, it is easy to verify that the numerical coefficients in (72) and (73) coincide with two significant digits.

Fig. 6 shows the dependence of the dimensionless damping rate $\gamma_q/\omega_0(q)$ at high temperatures region ($a \equiv (T/\nu_0 n_0) \gg 1$ and $T < T_c$) plotted for $F = 10^{-3}$. It is apparent that the graph plotted according to formula (71) or (72) (solid line) almost coincides with the graph, obtained by numerical computing based on formulae (50) - (52) (dashed line). As it follows from the plot at Fig. 4, the condition $(\gamma_q/\omega_0) \ll 1$ of the existence of sound in dilute gas with BEC is well satisfied up the region where $a \sim 30$. Naturally one should be confident that in a particular physical system the condition $T < T_c$ is satisfied too in this range of a .

V. CONCLUSION

Thus we have reported the main results relating to the construction of microscopic theory of sound attenuation due to Landau mechanism in dilute gases with Bose condensate. The analytical expressions of the propagation velocity and damping rate for sound in dilute gases with Bose condensate in the limiting cases of high and low

temperatures were obtained. Shown that at high temperatures these expressions coincide with those ones obtained previously by other authors in various phenomenological approaches. In the region of low temperatures the behavior of collisionless decrement of sound, obtained in the present paper, significantly differs from the same decrement, obtained by other authors. In our opinion the distinction is caused by our use of the evolution equations of investigated system that were obtained in the microscopic approach through the first principles.

We now make a significant remark. At first glance, the presence of the general equations (49) - (52) makes it possible to find the parameters of propagation and attenuation of sound in present system in any temperature range (i.e. for all values a), at least numerically. Meanwhile, Fig. 3 and Fig. 4 do not display data for the 'intermediate' temperature range, that is from $a = 0.7$ to $a = 1.4$. It is caused by the following circumstance. We have already mentioned that in this range of values of a there are no analytic methods for solving dispersion equations (49) - (52). But as it turned out, in this temperature range the numerical methods, at least those we have used, also become uncontrollable. Hence one cannot trust its results. This is supposed to be due the fact that in this interval of a the point $a \approx 0.847$ is situated. Recall that it is the point where the value of $s_0(a) = \omega_0(q, a)/u_0 q$ (see (49)) is equal to unity. As was previously mentioned the same value s gains also at the point $a = 0$ (zero sound). It is that very neighborhood of $a \approx 0.847$ that gives the huge oscillations of sound damping rate in the outcome of numerical computation with insignificant changes of value a . However, one cannot be sure that these oscillations of damping rate reflect a real physical picture. The reason for doubt in outcome is in mentioned uncertainty in this temperature range of the numerical methods used in the present paper. Numerical calculations are poorly controlled because of the slow convergence of the integrals in (50) - (52) due to the manifestation of non-analytic dependence on temperature of the dispersion characteristics of the system in the neighborhood of point $a \approx 0.847$. Apparently more detailed study of the behavior of decrement (or maybe even increment!) of sound in the neighborhood of this point requires the use of more sophisticated numerical methods. Authors intend to address this issue soon.

In this regard, the results in [32] are also of interest, where propagation and absorption of the transverse breathing mode of an elongated BEC were investigated experimentally in ^{87}Rb vapour. The authors of mentioned study attempting to measure the damping rate of these modes at the temperature approximately equal 40-60 nK had found that the behavior of the perturbation amplitude in this temperature region differs significantly from the behavior of the amplitude of a damped sinusoidal signal. The presence of such a phenomenon was suggested to be explained due to nonlinear effects in the propagation of studied modes in the system. We also would like to draw attention to the fact that the

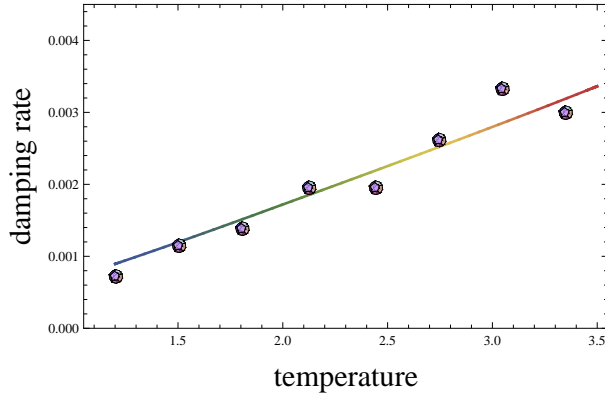


FIG. 7: Dependence of dimensionless damping rate γ_q/ω_0 on temperature (dimensionless quantity a). The solid line displays the result of numerical computation for $F = 3 \cdot 10^{-4}$ based on expressions (50)-(52). Diamonds reproduce experimental points taken from [32]

temperature range 40-60 nK corresponds to the values of quantity a in terms of the present article that are in

the 0.7 – 1.0 range. The value of a is naturally calculated according to formula (44) and using the values of the physical characteristics of the system [32]. In other words in the mentioned case [32] deals with the nearest neighborhood of the point $a \approx 0.847$ where nonanalytic dependence of the dispersion characteristics on temperature reveals.

In conclusion we note a good correlation between our theory in the case of high temperature region with the experimental data [32]. As can be seen from Fig.7, the results of numerical calculations based on formulae (50) - (52) for $F = 3.2 \cdot 10^{-4}$ (see(44)) are in satisfactory concordance with experimental data [32]. Experimental data reveals the same good fit with dependence given by formula (71).

The authors are naturally aware that a direct comparison of the results of the present paper with the experimental data [32] can hardly be considered as entirely correct. At least, the reason is that the authors of [32] have to deal with trapped BEC. However, such a comparison yet again demonstrates the validity of statement [12] that the linear dependence of the sound damping rate on temperature should occur in the case of trapped BEC.

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